

Lab 11 RLC Circuits

Purpose

To understand the phase shifts between the current and voltages in a RLC circuit and the electrical resonance phenomena.

Materials

Decade Resistance Box (CENCO), 0.1 μF , 0.5 μF , and 1.0 μF Capacitors, 2001/2001A Function Generator, GW INSTEK Oscilloscope (GOS-620), red and black cords, clamp clips, and coaxial heads.

Theory

When connected to an alternating current of frequency f , an inductor will have an inductive reactance

$$X_L = \omega L, \quad (1)$$

where $\omega=2\pi f$ is the angular frequency. A capacitor will have a capacitive reactance

$$X_C = \frac{1}{\omega C}. \quad (2)$$

In a RLC series circuit (see Figure 1), the total opposition to the current flow is given by the impedance

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \quad (3)$$

The impedance is at a minimum when

$$X_L - X_C = 2\pi fL - \frac{1}{2\pi fC} = 0. \quad (4)$$

This is also when the circuit current reaches its peak value. Solving for f gives the resonance frequency for this circuit

$$f_0 = \frac{1}{2\pi\sqrt{LC}}. \quad (5)$$

The resistor voltage has no phase shift from the current, which can be expressed as:

$$V_R(t) = \frac{R}{Z} V_{max} \sin(\omega t), \quad (6)$$

where V_{max} is the amplitude of the AC voltage from the function generator. The full voltage across the entire circuit has a phase shift with respect to the voltage across the resistor:

$$V_{in}(t) = V_{max} \sin(\omega t + \Phi), \quad (7)$$

where the phase shift is given by

$$\Phi = \tan^{-1} \left(\frac{X_L - X_C}{R} \right). \quad (8)$$

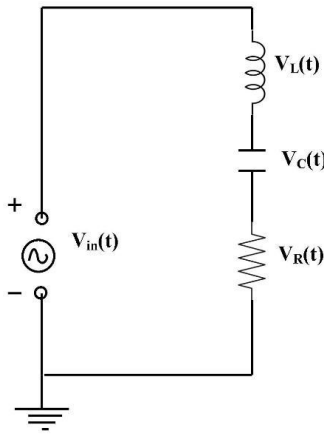


Figure 1. RLC circuit.

$V_{in}(t) = V_L(t) + V_R(t) + V_C(t)$. Equation (8) shows that if the capacitor is not connected in the circuit, then the phase shift in the RL circuit becomes positive (full voltage leads the current). If the inductor is not connected in the circuit, then the phase shift in the RC circuit becomes negative (full voltage lags behind the current). If X_L is greater than X_C , the phase Φ is positive. If X_L is less than X_C , the phase Φ is negative.

The phase of the capacitor voltage lags behind the resistor voltage by 90° :

$$V_C(t) = \frac{X_C}{Z} V_{max} \sin(\omega t - \frac{\pi}{2}). \quad (9)$$

The phase of the inductor voltage leads the resistor voltage by 90° .

$$V_L(t) = \frac{X_L}{Z} V_{max} \sin(\omega t + \frac{\pi}{2}). \quad (10)$$

The graphic representation of the phase differences among the three voltages is shown in Figure 2.

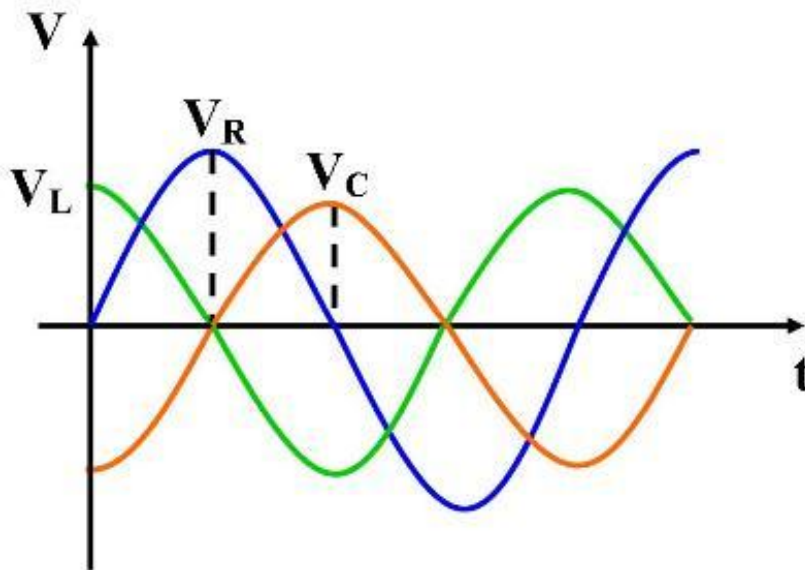


Figure 2. Phase shifts of the voltages across an inductor and a capacitor.

If we apply different voltage signals to the horizontal and vertical inputs of an oscilloscope, the phase differences will produce an elliptical pattern on the screen, which is a Lissajous curve. Figure 3 shows some Lissajous curves and the phase angles ϕ , where $x = A \sin(\omega t + \phi)$ and $y = B \sin(\omega t)$. A and B

represent the voltage amplitudes from CH1 and CH2, respectively. Lissajous curves can become a line with a positive slope when ϕ is zero.

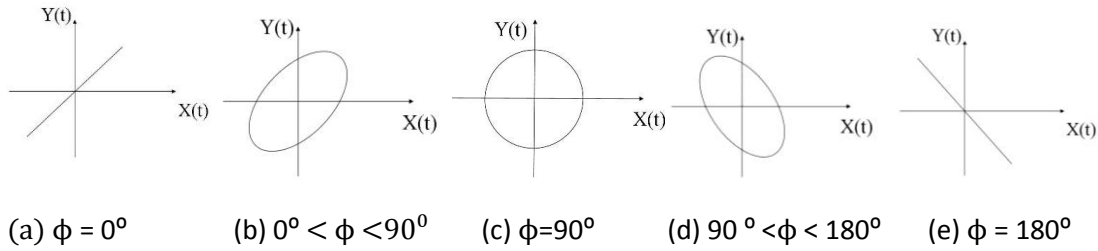


Figure 3. Some Lissajous curves.

Procedures:

1. Observe sinusoidal signal from the function generator.

For 2001 model, use the connector “HI” (600 Ohm), adjust “AMPLITUDE” so that voltage is 3 V. For 2001A model, connect cable to “OUTPUT 600 Ohms”, set the RANGE at “1-10 V”, and adjust the “FINE” knob to get the peak-to-peak voltage signal 3 V (peak-to-peak voltage is the algebraic difference between the highest voltage and the lowest voltage in the signal). Set frequency to 800 Hz, and press the sinusoidal wave button.

Set *VERTICAL* AC-GND-DC switches to AC for both CH1 and CH2. Next, set the *VERTICAL* MODE switch to CH1. Connect CH1 from the oscilloscope to the function generator to observe the full voltage from the function generator. Set *TRIGGER* MODE and *SOURCE* to AUTO and CH1, respectively. Be sure that the red lead from the function generator is connected to the red probe lead of the oscilloscope (see Image 1 or 2). The ground (black) lead from the function generator is connected to the ground (black) probe lead of the oscilloscope. Turn the *TIME/DIV* knob so that about one and a half full oscillation cycles can be seen on the screen.

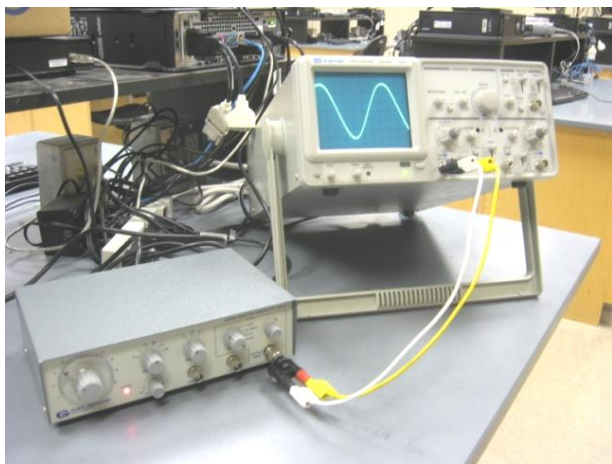


Image 1

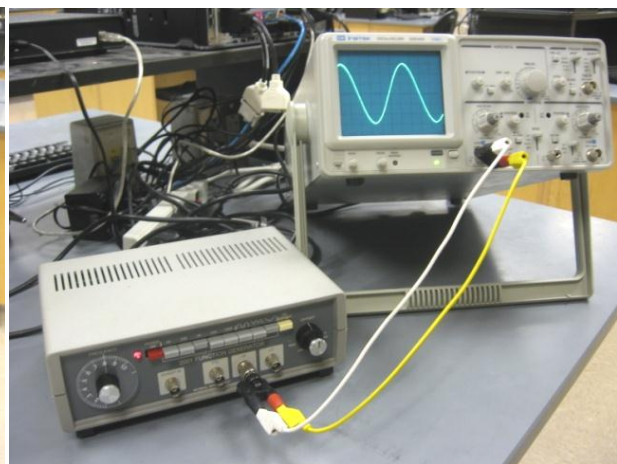
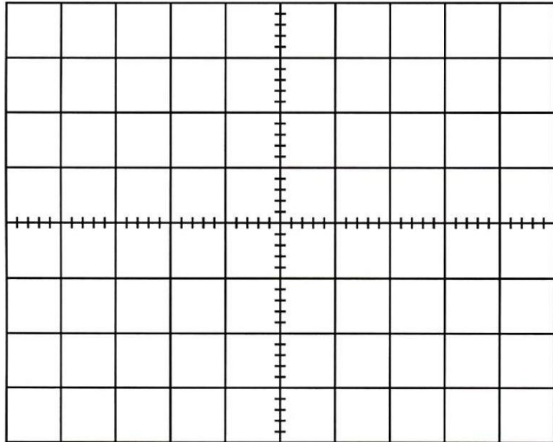


Image 2

Draw the oscilloscope display on the graticule provided below:



TIME/DIV= _____ VOLTS/DIV= _____

2. Observe the voltage across a resistor connected to the function generator in a closed circuit.

Set the Resistance Box to $600\ \Omega$ and connect the resistance box terminals to the red and black leads of the function generator to form a closed circuit. Connect the red and black probe leads of CH1 in the oscilloscope to the red and black leads of the function generator, respectively (see Image 3). The circuit connection is the same for the 2001 model function generator in this step as well as all the following steps (although the images only show 2001A model). Draw the oscilloscope display on the graticule provided below.

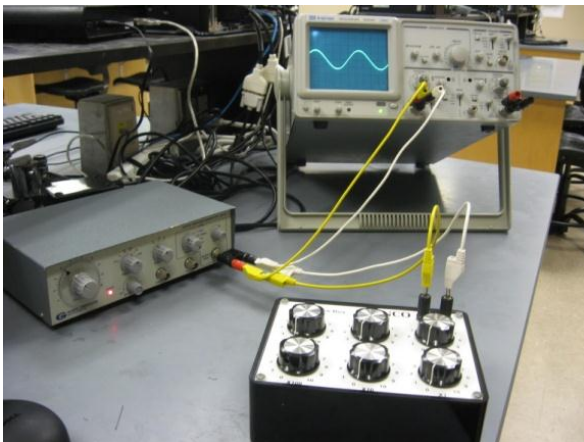
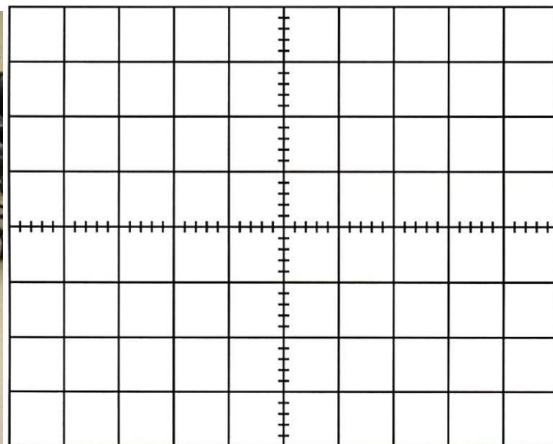


Image 3



TIME/DIV= _____ VOLTS/DIV= _____

Compare and contrast the scope displays in Steps 2 and 1:

- Observe the full voltage across a RC series circuit in comparison to the voltage across the resistor.

Connect the capacitor and the resistance box with the function generator in a series circuit (see Figure 4 and Images 4 and 5). The resistance box is still set at $600\ \Omega$, and the capacitor used is $1\ \mu\text{F}$. Be sure to connect the black probe leads (ground) of both CH1 and CH2 to the ground of the function generator! Connect the red probe lead from CH1 to the higher voltage potential terminal of the capacitor (the terminal which is connected to the red lead of the function generator). Connect the red probe lead of CH2 to the higher voltage potential terminal of the resistance box (the terminal which is connected to the capacitor). Make sure CH2 INV is not pressed in. Switch *VERTICAL MODE* to DUAL. CH1 and CH2 VOLTS/DIV settings should be equal.

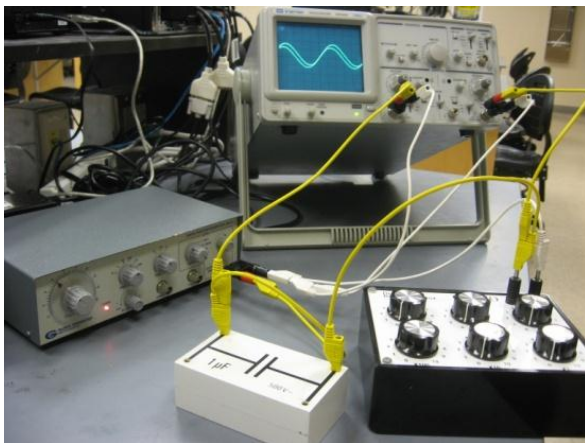


Image 4

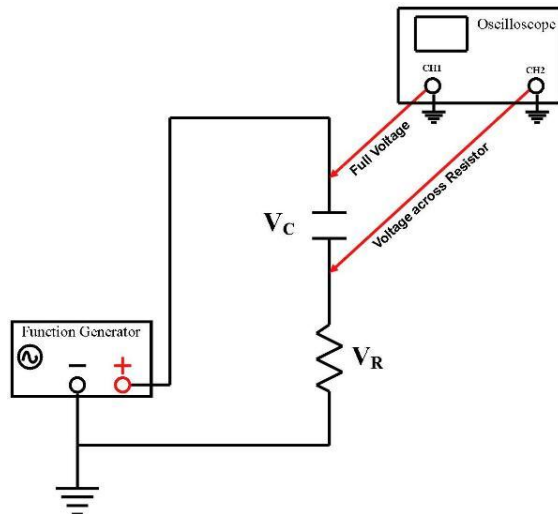
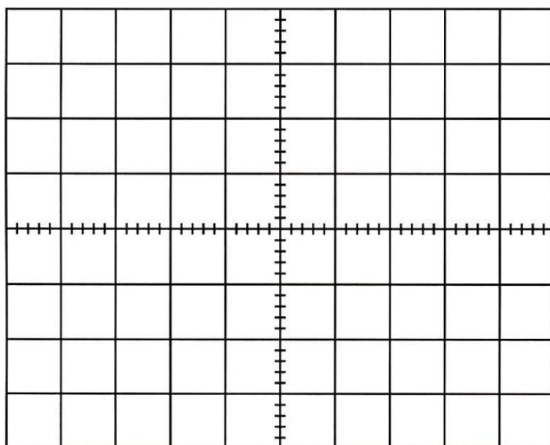


Figure 4. RC circuit and oscilloscope.

Draw the oscilloscope display on the graticule provided below and label the voltage signals (“ $V_R(t)$ ” or “ $V_{RC}(t)$ ”). Describe the phase difference between the two waveforms on the oscilloscope display. Infer if the capacitor has made the phase of the full voltage lag behind the resistor voltage. (Hint: Scanning from the left of the scope display, the waveform with its peak arriving first leads in the phase.)



TIME/DIV= _____ VOLTS/DIV= _____



Image 5

- Observe the full voltage in a RL series circuit in comparison to the voltage across the resistor.

Connect an inductor (around 300 mH) and the resistance box to the function generator in a closed RL series circuit. Use the clamp clips to connect the terminals of the inductor (see Figure 5 and Image 6).

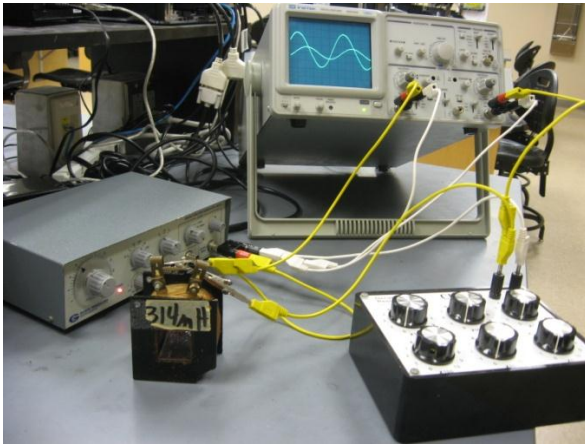


Image 6

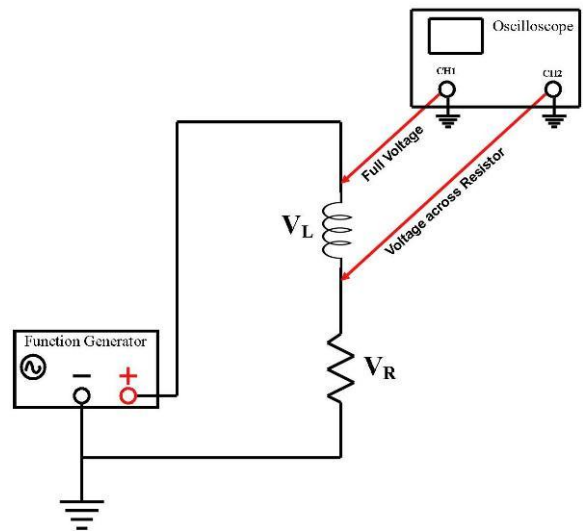
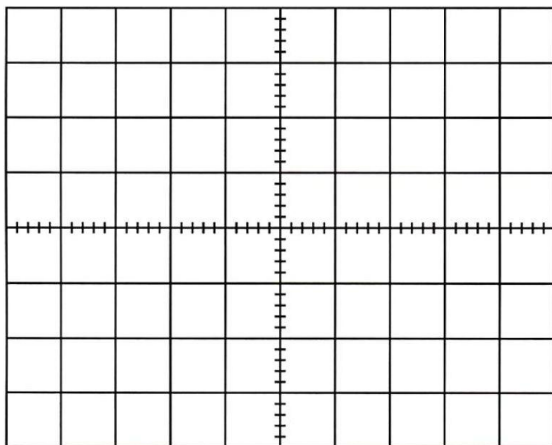


Figure 5. RL circuit and oscilloscope.

Draw the oscilloscope display on the graticule provided below and label the voltage signals (“ $V_R(t)$ ” or “ $V_{RL}(t)$ ”). Describe the phase difference between the two waveforms on the oscilloscope display. Infer if the inductor has made the phase of the full voltage lead the resistor voltage.



TIME/DIV= _____ VOLTS/DIV= _____

- Observe phase difference between the full voltage across all three elements and the voltage across the resistor in a RLC series circuit. Connect the inductor, capacitor and resistance box to the function generator in a RLC series circuit (see Image 7 and Figure 6).

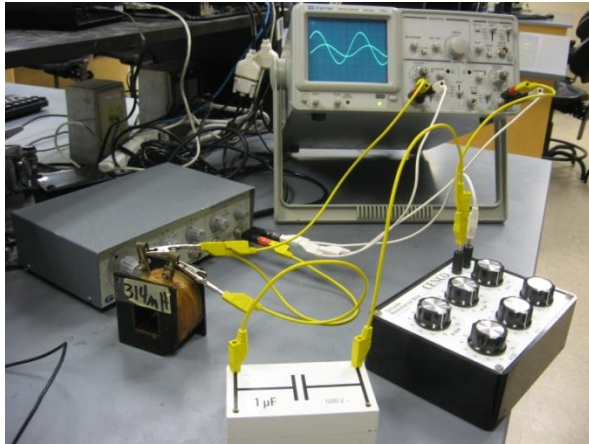


Image 7

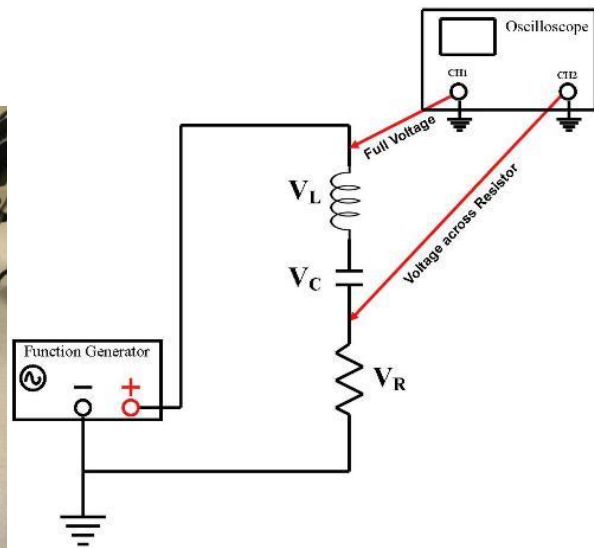


Figure 6. RLC circuit and oscilloscope.

- Observe Lissajous curves and find the resonance frequency in RLC series circuit.

Turn the TIME/DIV knob to X-Y position. CH1 is utilized as the horizontal deflection (X-axis) while CH2 is utilized as the vertical deflection (Y-axis). An elliptical curve appears on the screen, showing that voltages measured in CH1 and CH2 are out of phase (see image 8). Specifically, it shows that the full voltage (X-axis) is leading the resistor voltage (Y-axis), as illustrated in Figure 3(b).

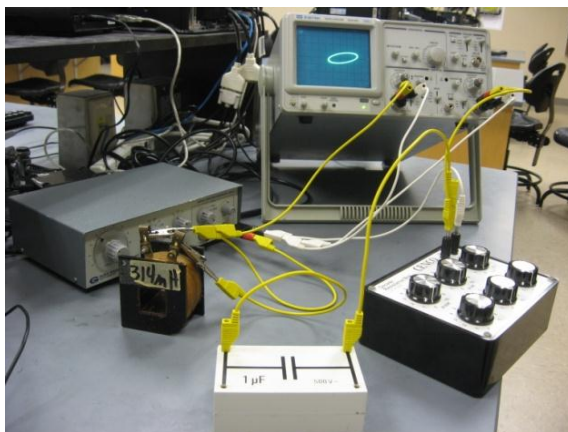


Image 8

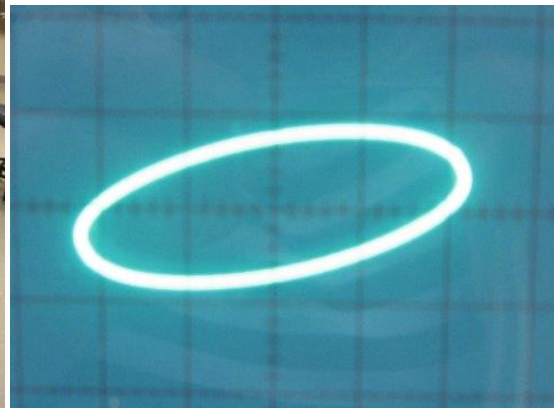
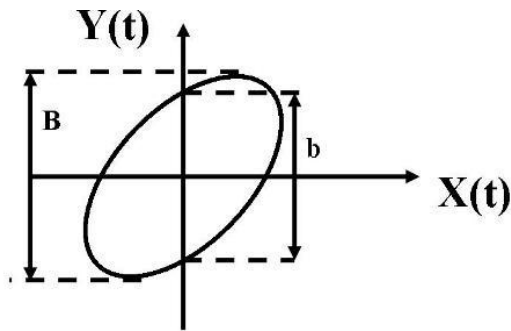


Image 9

To determine what the phase difference ϕ is, one can position the Lissajous curve to the center of the screen (see Image 9). Read the lengths of B and b as illustrated in Figure 7, where B is the length between two tangent lines to the curve at the top and the bottom; b is the length between

the two y-intercepts. Then, ϕ can be calculated as $\phi = \sin^{-1} \left(\frac{b}{B} \right)$. For example, assume $b=1.5$



divisions, $B=1.6$ divisions, then $\phi = \sin^{-1} \left(\frac{1.5}{1.6} \right) = \sin^{-1} \left(\frac{1.5}{1.6} \right) = 69.6^\circ$.

Record your b and B below:

$B =$ _____

$b =$ _____

$\phi =$ _____

Figure 7. Phase difference between $X(t)$ and $Y(t)$.

Theoretically, we can calculate the phase difference as follows:

$$X_L = 2\pi fL = 2 * 3.14 * 800 * 314 * 10^{-3} = 1.58 * 10^3 \Omega.$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2 * 3.14 * 800 * 1.0 * 10^{-6}} = 2.0 * 10^2 \Omega.$$

$$\text{Therefore, } \phi = \tan^{-1} \left(\frac{X_L - X_C}{R} \right) = \tan^{-1} \left(\frac{1.58 * 10^3 - 2.0 * 10^2}{600} \right) = 66.5^\circ.$$

Turn the frequency knob on the function generator down till the Lissajous curve becomes a straight line (see Image 10). Record this frequency. This is also the frequency that makes the impedances of the inductor and capacitor equal so that they cancel each other due to opposite phases of their voltages.

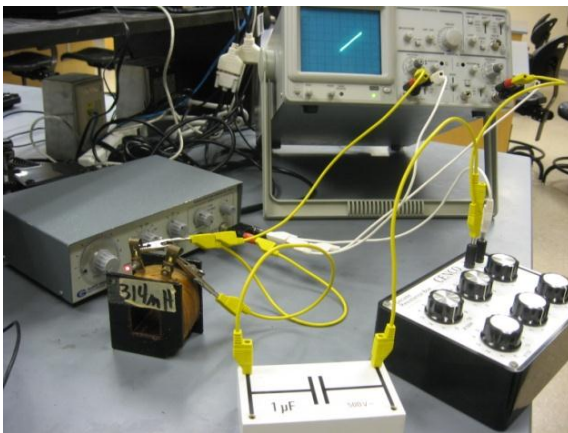


Image 10

Observed resonance frequency = _____ Hz

Theoretical resonance frequency = _____ Hz

(Hint: For example, the theoretical resonance frequency can be calculated as shown below:

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{314 * 10^{-3} * 1.0 * 10^{-6}}} = 284 \text{ Hz})$$

Percentage error = _____

7. Observe the impact of a different capacitor on the resonance frequency in a RLC series circuit.

Connect a $0.5 \mu\text{F}$ capacitor in the RLC circuit as in Step 6 (see Image 11). Record the resonance frequency for the new RLC circuit below:

Observed resonance frequency = _____ Hz

Theoretical resonance frequency = _____ Hz

Percentage error = _____

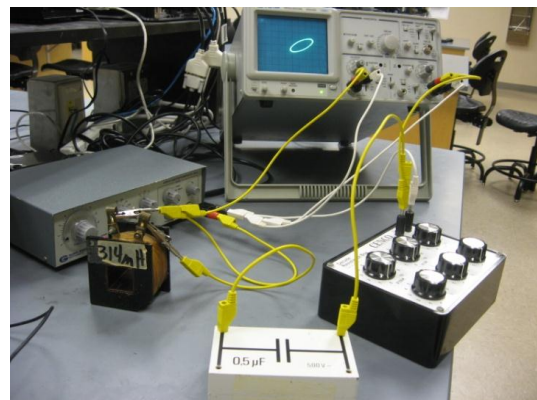


Image 11

Connecting question:

1. If you were to replace the $0.5 \mu\text{F}$ capacitor with a $0.1 \mu\text{F}$ one in the RLC circuit described in Step 7, how would you predict the resonance frequency in the RLC circuit would change?

2. Compare and contrast mechanical resonance with electrical resonance in the RLC circuit.