INTRODUCTION

Purpose

The purpose of this lab is two-fold:

1. to provide you with our expectations for the semester and
2. to introduce you to some of the analytical tools we'll use this semester to analyze the results of our experiments.

Lab safety

This laboratory does not require the use of personal protective equipment but we will enforce safe lab practices. In particular, there is no eating or drinking permitted in the laboratory. Safety issues will be identified at the beginning of each laboratory session by your instructor. Repeated or serious violation of safe lab practices will be cause for dismissal from the laboratory.

! The most important laboratory safety practice is to wash your hands thoroughly at the end of each laboratory session.

Lab handouts

Lab handouts can be found on the lab web site. You should review and print a copy of the handout *before* coming to class. Handouts will not be provided during class.

Completed lab handouts will constitute your laboratory reports. These are due at the end of the laboratory session from each individual; group reports will not be accepted. An example of a well-written lab handout is provided on the course web site.

! Particularly important sections of the lab handouts will be identified by the large exclamation point in the margin, like the one in front of this section.

? Places where you will be expected to provide an answer will be identified by the large question mark in the margin, like the one that precedes this section. In answering questions in the handout, be succinct and be quantitative. The answers to questions will *never* be just "yes" or "no." Responses should be short; limit yourself to one or two sentences. Note also that the word "data" is a plural noun and takes the plural verb tense. For example: the data *have* a mean value of 6.3 cm.

Places where you are expected to generate a plot for submission with your lab handout will be identified with the plot icon that identifies this section. Generally, we want to limit paper consumption, so export plots from Data Studio to the My Documents directory and paste two or more plots onto a single page of a Word document. The plots will be attached to the lab handout and submitted at the end of the lab.

Places where you are expected to include a table of your results for submission with your lab handout will be identified with the table icon that precedes this section. Again, do not just print the table from Excel or Data Studio. Paste the relevant data into a Word document. Tables submitted with your lab handout must be formatted to the appropriate number of significant figures.

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Final examination

The lab final will be conducted during the final laboratory session. It is a practical examination that will test your ability to conduct an experiment like those performed during the semester, analyze the results using the computational tools and present the results in a systematic fashion.

Units

The measurements we make this semester will, in almost all cases, involve units of measure. We will also make extensive use of computers to perform data analysis. The computers will calculate for you but they will not *think* for you. You will be responsible for insuring that your units are consistent throughout your calculations.

Table 1. Common SI units.

property	**unit**	**symbol**
distance	meter	m
time	second	s
mass	kilogram	kg
velocity		m/s
acceleration		m/s^2
force	Newton	N ($= kg{\cdot}m/s^2$)
energy	Joule	J ($= kg{\cdot}m^2/s^2$)
power	Watt	W ($= kg{\cdot}m^2/s^3$)
pressure	Pascal	Pa ($= kg/m{\cdot}s^2$)

Note that, strictly speaking, it is not a units conversion to move between centimeters (cm) and meters (m). Both of these quantities have the same base unit: meters. The c represents one of twenty SI prefixes, which provide a compact representation of scientific notation. The complete prefix list is shown in the table below.

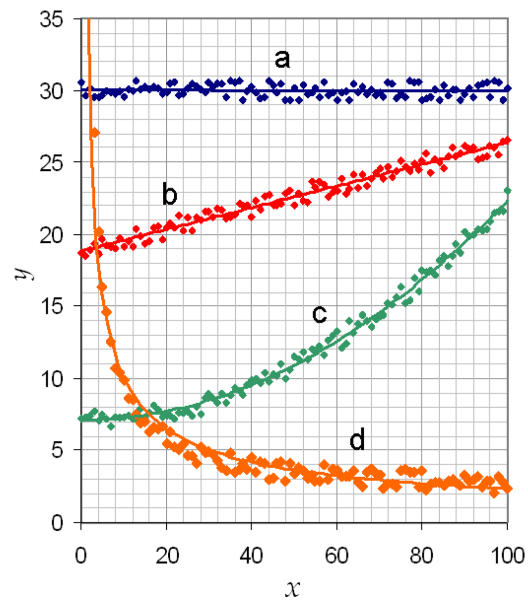
Table 2. SI prefixes. Note that the prefixes are case sensitive.

prefix	**symbol**	**value**	**prefix**	**symbol**	**value**
deci	d	10^{-1}	deka	da	10^{1}
centi	c	10^{-2}	hecto	h	10^{2}
milli	m	10^{-3}	kilo	k	10^{3}
micro	μ	10^{-6}	mega	M	10^{6}
nano	n	10^{-9}	giga	G	10^{9}
pico	p	10^{-12}	tera	T	10^{12}
femto	f	10^{-15}	peta	P	10^{15}
atto	a	10^{-18}	exa	E	10^{18}
zepto	z	10^{-21}	zetta	Z	10^{21}
yocto	y	10^{-24}	yotta	Y	10^{24}

Measurements and Errors

Measurements are characterized by their *precision* and *accuracy*. Precision is a statement of the number of significant figures in the measurement, while accuracy can be thought of as the closeness of the result to the "correct" answer. One can, for example, make an extremely precise but inaccurate measurement. This can happen if there are systematic biases in the measurement system. Because measurements have finite precision, experiments never "prove" a result; we'll learn how to state the results of our experiments.

Consider the four sets of measurements displayed in the figure below. For the upper trace, denoted a in the figure, the data do not have any obvious dependence on the x-axis variable. They do, however, appear to scatter around a particular y-value.



For data that should, or at least do, cluster around a particular value, the data points y_i can be described by a series of moments. The first moment is called the mean; it is denoted by $\bar{y}$ and defined as follows:

$$\bar{y} = \frac{1}{N}\sum_{i=1}^{N} y_i, \tag{1}$$

where the greek symbol sigma indicates summation and there are N measurements y_i.

The second moment is the variance, defined below:

$$\mathrm{var}(y_1, \ldots, y_N) = \frac{1}{N-1}\sum_{i=1}^{N} (y_i - \bar{y})^2 . \tag{2}$$

The standard deviation $\sigma(y)$ is defined as follows: $\sigma(y) = \sqrt{\mathrm{var}(y_1, \ldots, y_N)}$. We will always use the standard deviation $\sigma(y)$ to describe our results because it has the same units as the mean $\bar{y}$. We'll talk more about the standard deviation in a moment but in this course we'll use it to quantify the uncertainty in our measurements. For example, the mean value of the points in curve a is $\bar{y} = 29.96$ with a standard deviation of $\sigma(y) = 0.41$. For a smaller standard deviation, the scatter in the data would be reduced; a larger standard deviation would signify a greater scatter in the data.

Trends in the data

Now consider the other curves b, c and d in the figure. Each of these demonstrates a definite dependence of the data on x. We would say that each curve depicts a *trend* in the data. None of these curves can be adequately represented by a mean value $\bar{y}$ and a standard deviation $\sigma(y)$. Instead, we will want to identify a more accurate description of the data.

For example, the data in curve b follow a linear trend. That is, we would expect to be able to describe the bulk of the x-dependence of those data with a function of the form $y = mx + b$. Without going into the numerical details, fitting a line to curve b will result in estimates of

$m \pm \sigma(m)$ for the slope and $b \pm \sigma(b)$ for the y-intercept, where the scatter in the data result in the uncertainties $\sigma(m)$ and $\sigma(b)$ of our estimates m and b. For the data in curve b, fitting a line to the data yields the following estimates: $m = 0.0694 \pm 0.0014$ and $b = 19.090 \pm 0.083$. Note that we haven't specified units for any of these example values. In our labs, almost all quantities will have units; it will be incorrect to omit units.

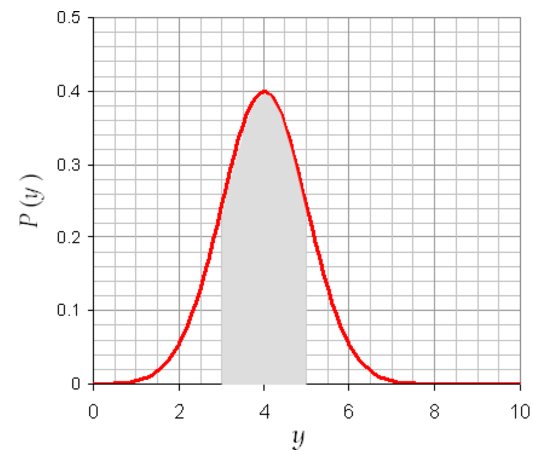
The data in curve c appear to depend quadratically on x. That is, we would seek to find a function $y = ax^2 + bx + c$ to describe the data. For these data, the fit yields the following estimates: $a = 0.001411 \pm 0.000053$, $b = 0.0101 \pm 0.0055$ and $c = 6.92 \pm 0.11$.

Similarly, the data in curve d appear to depend inversely on x. We might expect that a function $y = a/x + b$ would describe these data. Fitting such a curve yields the following estimates: $a = 75.12 \pm 0.40$ and $b = 1.995 \pm 0.051$.

Now how are we to interpret the uncertainties σ? We will make the usual simplifying assumption that our errors are normally distributed. That is, that the probability $P(y)$ of making a measurement with a value y is given by

$$P(y) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(y-\bar{y})^2/2\sigma}. \tag{3}$$

A plot of a normal distribution is illustrated in the figure at right. The mean value $\bar{y}$ in this case was 4 and the standard deviation σ was 1.



The total area under the curve is one; that is, the result of any measurement is guaranteed to be some value y. If we make many measurements and then ask what is the probability that the results are within one standard deviation σ of the mean $\bar{y}$ (the area of the shaded region), the answer is 0.68. Similarly, we would find that 0.95 of the results lie within 2σ of the mean and 0.997 of the results lie within 3σ of the mean.

Analysis and Interpretation of Results

! We will use the following definitions in this laboratory:

- There are two significant figures in σ. All results will be reported to this precision.
- To compare two numbers, subtract and divide by σ. Measurements that are within $\pm 3\sigma$ of some expected result are deemed *consistent*. Measurements beyond 3σ are *inconsistent*.

? How many significant figures are there in the estimates for the coefficients m and b in the linear fit to curve b? How many significant figures are there in the estimates for the coefficients a, b and c in the quadratic fit to curve c?

To illustrate the second definition, consider that the "correct" value for the data in curve a is 30. Is our estimate of $y = 29.96 \pm 0.41$ consistent with that value? Most students would immediately say that it is obvious: 29.96 is really close to 30. In this laboratory, we want to quantify "really close," so we compute the following:

$$\frac{y - \bar{y}}{\sigma} = \frac{30 - 29.96}{0.41} \approx 0.1.$$

Because $|0.1| < 3$, we conclude that our measurements are consistent with the expected value. Note that because σ has the same units as y, the quantity $(y - \bar{y})/\sigma$ is *dimensionless*. By convention, we would report that $\bar{y}$ is within 1σ of y.

Data Studio

We will use two computer analysis tools this semester. The first is Data Studio. If you have not already done so, log into the computer and open the lab web site. Download the files listed under Materials to the My Documents directory. Open the Data Studio application on the Desktop and then close the popup window that displays.

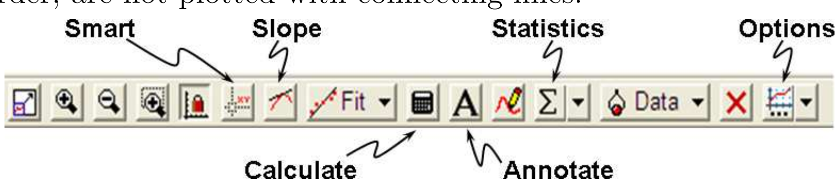
We will often use Data Studio as a data acquisition tool but for today, we'll use it just for analysis.

1. Select File from the task bar and then Import.
2. Select the intro1.txt file as the data source.

This will provide an initial data set for us to analyze. First, let's plot the data.

3. Click and hold on the Data icon (upper left corner of the window). Drag to the Graph icon in the lower portion of the window and release.

This will produce a graph of our data. The graph toolbar is illustrated below. As a general rule, data that are not generated by means of a device that draws lines, such as a strip chart recorder, are not plotted with connecting lines.



4. To remove the connecting lines, select the Options (rightmost) icon in the Graph toolbar. Turn off the connecting lines in the pulldown menu.

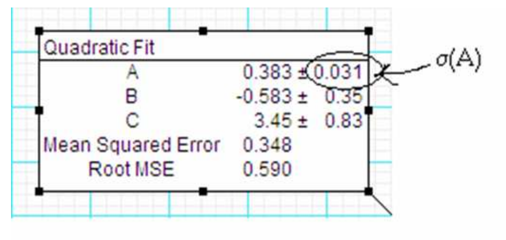
In most of our experiments, we will be seeking to verify a model that is a function of some parameters defining the system. This means that we will want to take the given model and adjust the parameters to obtain some sort of best fit. The three things we would like to obtain from such a process are:

- the estimated parameters,
- error estimates on the parameters and
- some estimate of how well the model describes the data.

This last estimate is often difficult to obtain; the usual goodness-of-fit measures require somewhat circular reasoning for their validation. Most often, visual inspection of a plot of the data and fitted curve will be a good indication of how well the model describes the data.

5. Fit a quadratic function to the data by clicking on the Fit button on the Graphics toolbar and selecting Quadratic.

Data Studio fits a function of the form $y = Ax^2 + Bx + C$ to the data and reports values for A, B and C and their standard deviations $\sigma(A)$, $\sigma(B)$ and $\sigma(C)$. You should see a panel appear similar to the one shown at right.



? Note that the value for A is reported according to the laboratory standard: there are two significant significant figures in $\sigma(A)$ and A is reported to the same number of decimal places as $\sigma(A)$. Does Data Studio report the other values properly? If not, present them properly below.

6. Fit a linear function to the data.

? Which of the two functions, linear or quadratic, is a better fit to the data? Justify your answer.

Paste the graph into a Microsoft Word document for submission with your lab.

7. Import the second set of data from the file intro2.txt to a new table. We can use tools from the Graph toolbar to analyze the data. For example, the Smart tool will report the values of individual data points.
8. Select a portion of the initial flat section of the data by clicking and holding on the graph and dragging. A rectangle will appear that denotes your selection. When you release the mouse, the selected data will appear with yellow highlights.
9. Compute the mean and standard deviation of these data by using the Statistics icon. To adjust the number of decimal places reported by the Statistics tool, double click the Table icon in the Data window. Select the Numeric tab. Change the number of decimal places as necessary and turn off the Scientific Notation Threshold.

? What values do you obtain for the mean and standard deviation of the flat section?

10. Now select data from the upper flat portion of the plot.

? What values do you obtain for the mean and standard deviation?

11. Fit a linear function to the central section of the data.

? What value do you obtain for the slope? (Don't forget to report the standard deviation!)

Excel

We will use Excel as our other analysis tool this term. Download the workbook for this lab from the course web site to your My Documents directory if you haven't already done so and open it in Excel. DO NOT open the workbook in the browser.

Cells in the worksheet are indexed by a column letter and row number like D9 or I2. Note that cells in the worksheet can contain numeric data or alphanumeric strings. They can also contain formulas, which are prefaced by an equal sign (=).

Basic Mathematical Operations

Excel uses the symbols shown at right for executing basic mathematical operations. Operations can be grouped with parentheses (); otherwise, Excel will perform exponentiation first, then either multiplication or division and finally addition or subtraction. One must be explicit in defining formulas because Excel will simply do what you tell it to do, not necessarily what you want it to do.

Operation	**Symbol**
addition	+
subtraction	−
multiplication	*
division	/
exponentiation	^

1. Go to cell B5 on the Practice worksheet and type =a5/1000<enter>

When you have finished, Excel should display the value 0.01 in the cell.

2. Click on cell B5 again and notice that the formula that you typed is now displayed in the formula bar (denoted by the fx). Click in the formula bar and notice that the cell index A5 turns color and that the border for cell A5 turns the same color. This will be a useful debugging tool.
3. Now hit <escape> and click in cell B5 again. Move the cursor to the lower right hand corner of the cell until it changes into a black cross. Now click and hold the left mouse button and drag down to cell B9. Release the mouse button.

When you release the button, Excel will have replicated the formula in cell B5 through the range down to B9.

! When copying cells, Excel will normally increment the indices, depending on the direction of motion. Dragging or copying down will increment the row index. Dragging or copying across will increment the column index. In order to prevent Excel from incrementing the index, we have two options:

- Pin the cell index by using a $. For example, $c3 would allow the row index (3) to change but prevents the column index (C) from changing. c$3 fixes the row index but permits the column index to change and c3 prevents either index from changing.

- Name the cell via Insert. . .Name. . .Define.

4. Let us name the cell B2. Click in the cell and choose Insert..Name..Define. Excel will suggest a name based on the information found in neighboring cells. Define B2 to be the name g.
5. Click on cell E5 and type =. Then click on cell B5 and type *g<enter>. (One can either type the cell index or just click on the cell; either method is acceptable to Excel.)

Excel should display 0.098. Click on cell E5 again and then click in the formula bar. Note that both B5 and g have turned colors and the respective cell borders are colored to match. Hit <escape>.

6. Replicate the formula for cells E6..E9. (Click and drag; DO NOT retype.)
7. Complete the worksheet.

Paste the worksheet into the Microsoft Word document along with your plot for submission with the lab.

? The last column of the worksheet shows the comparison between the weight W and the product mg. With the data provided, is the weight consistent with the product mg? Justify your answer; be quantitative and succinct.

Making Connections

Suppose that the physical model we were investigating when we generated the data in intro1.txt was defined by the following equation:

$$x = x_0 + v_0 t + \tfrac{1}{2} a t^2.$$

We used the quadratic function to fit our results. To what do the coefficients A, B and C correspond, if our x values in the table are time and the y values are displacement?

The figure below shows the graph of velocity versus time for an object moving with constant acceleration. The slope of the graph gives a measure of acceleration. Is the experimental result consistent with the accepted value of 0.375 m/s^2? Justify your answer.